AP Calculus AB
Review 1, No Calculator
Name: $\qquad$
Complete all the following on notebook paper. *Solutions follow this B 1. page
Which of the following defines a function $f$ for which $f(-x)=-f(x)$ ?
(A) $f(x)=x^{2}$
(B) $f(x)=\sin x$
(C) $f(x)=\cos x$
(D) $f(x)=\log x$
(E) $f(x)=e^{x}$

C 2 .
$\ln (x-2)<0$ if and only if
(A) $x<3$
(B) $0<x<3$
(C) $2<x<3$
(D) $x>2$
(E) $x>3$
$B 3$.
If $\left\{\begin{array}{l}f(x)=\frac{\sqrt{2 x+5}-\sqrt{x+7}}{x-2}, \text { for } x \neq 2, \\ f(2)=k\end{array}\right.$ and if $f$ is continuous at $x=2$, then $k=$
(A) 0
(B) $\frac{1}{6}$
(C) $\frac{1}{3}$
(D) 1
(E) $\frac{7}{5}$
$\pm 4$
$\int_{0}^{8} \frac{d x}{\sqrt{1+x}}=$
(A) 1
(B) $\frac{3}{2}$
(C) 2
(D) 4
(E) 6

Es.
If $3 x^{2}+2 x y+y^{2}=2$, then the value of $\frac{d y}{d x}$ at $x=1$ is
(A) -2
(B) 0
(C) 2
(D) 4
(E) not defined
$B 6$
What is $\lim _{h \rightarrow 0} \frac{8\left(\frac{1}{2}+h\right)^{8}-8\left(\frac{1}{2}\right)^{8}}{h}$ ?
(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) The limit does not exist.
(E) It cannot be determined from the information given.

D7.

For what value of $k$ will $x+\frac{k}{x}$ have a relative maximum at $x=-2$ ?
(A) -4
(B) -2
(C) 2
(D) 4
(E) None of these

Br.
If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$
(A) 2
(B) 3
(C) 6
(D) 11
(E) 13

C 9.
When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is
(A) $\frac{1}{4 \pi}$
(B) $\frac{1}{4}$
(C) $\frac{1}{\pi}$
(D) 1
(E) $\pi$
$E_{10}$
The set of all points $\left(e^{t}, t\right)$, where $t$ is a real number, is the graph of $y=$
(A) $\frac{1}{e^{x}}$
(B) $e^{\frac{1}{x}}$
(C) $x e^{\frac{1}{x}}$
(D) $\frac{1}{\ln x}$
(E) $\ln x$
11. 2000—AB4

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \leq t \leq 120$ minutes. At time $t=0$, the tank contains 30 gallons of water.
(a) How many gallons of water leak out of the tank from time $t=0$ to $t=3$ minutes?
(b) How many gallons of water are in the tank at time $t=3$ minutes?
(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time $t$.
(d) At what time $t$, for $0 \leq t \leq 120$, is the amount of water in the tank a maximum? Justify your answer.
12. $200-\mathrm{AB} 5$

Consider the curve given by $x y^{2}-x^{3} y=6$.
(a) Show that $\frac{d y}{d x}=\frac{3 x^{2} y-y^{2}}{2 x y-x^{3}}$.
(b) Find all points on the curve whose $x$-coordinate is 1 , and write an equation for the tangent line at each of these points.
(c) Find the $x$-coordinate of each point on the curve where the tangent line is vertical.

Review 1
1.

$$
\begin{aligned}
& \rightarrow f(x)=\sin x \\
& f(-x)=\sin -x \\
& e x f\left(-\frac{\pi}{2}\right)=\sin \left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{2}\right) \\
&=-\sin x \\
&=-1
\end{aligned}
$$

2. In $x<0$ when $0<x<1 \therefore 0<(x-2)<1$ when

$$
2<x<3 \quad C
$$

3. $f(x)=\frac{\sqrt{2 x+5}-\sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2 x+5}+\sqrt{x+7}}{\sqrt{2 x+5}+\sqrt{x+7}}$

$$
\begin{aligned}
& =\frac{2 x+5-(x+7)}{(x-2)(\sqrt{2 x+5}+\sqrt{x+7})}=\frac{x-12}{x / 2 \sqrt{2 x+5}+\sqrt{x+7}} \\
& \lim _{x \rightarrow 2} H(x)=\frac{1}{6} \\
& k=\frac{1}{6} \quad B
\end{aligned}
$$

$4 . \int_{0}^{8} \frac{d x}{\sqrt{1+x}} u=1+x \quad d u=d x$

$$
\int_{1}^{9} \frac{d u}{\sqrt{u}}=\int_{1}^{9} u^{-1 / 2} d u=\left.2 u^{1 / 2}\right|_{1} ^{9}=2 \sqrt{9}-2 \sqrt{1}=6-2=4
$$

5. $\quad 6 x+2 x \frac{d y}{d x}+2 y+2 y \frac{d y}{d x}=0$
$d y=-6 x-2 y$

$$
d x \quad 2 x+2 y
$$

$$
=\frac{-6(1)-2(-1)}{2(1)+2(-1)}=\frac{-6+2}{2-2}
$$

$$
\begin{aligned}
& 3(1)^{2}+2(1) y+y^{2}=2 \\
& y^{2}+2 y+1=0 \quad \frac{(y+1)(y+1)}{y=-1}
\end{aligned}
$$

6. 

$$
\begin{aligned}
& f(x)=8 x^{8} \\
& f^{\prime}(x)=64 x^{7} \\
& f^{\prime}\left(\frac{1}{2}\right)=64\left(\frac{1}{2}\right)^{7}=\frac{64}{1-8}=\frac{1}{2} \quad
\end{aligned}
$$

7. 

$$
\begin{aligned}
& f(x)=x+\frac{k}{x} \\
& f^{\prime}(x)=1-\frac{k}{x^{2}} \\
& f^{\prime}(-2)=0=1-\frac{k}{4} \rightarrow \frac{k}{4}=1 \rightarrow k=4 D
\end{aligned}
$$

8. 

$$
\begin{aligned}
& P(x)=(x+2)(x+k) \\
& P(1)=12=(1+2 x 1+k) \\
& 12=3(1+k) \\
& 12=3+3 k \\
& k=3
\end{aligned}
$$

9

$$
\begin{aligned}
& A=\pi r 2 \quad \frac{d A}{d t}=2 \frac{d r}{d t} \\
& \frac{d A}{d t}=2 \pi r \frac{d r}{d t} \\
& p d t=2 \pi r d r \\
& 1=\pi r \\
& r=\frac{d}{\pi} C
\end{aligned}
$$

10. 

$$
\begin{aligned}
& x=e^{t} \quad y=t \\
& x=e^{y} \\
& \ln x=\ln e^{y} \rightarrow \ln x=y \quad E
\end{aligned}
$$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \leq t \leq 120$ minutes. At time $t=0$, the tank contains 30 gallons of water.
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(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time $t$.
(d) At what time $t$, for $0 \leq t \leq 120$, is the amount of water in the tank a maximum? Justify your answer.
(a) Method 1: $\int_{0}^{3} \sqrt{t+1} d t=\left.\frac{2}{3}(t+1)^{3 / 2}\right|_{0} ^{3}=\frac{14}{3}$

- or -


## Method 1:

Method 2: $L(t)=$ gallons leaked in first $t$ minutes

$$
\begin{aligned}
\frac{d L}{d t} & =\sqrt{t+1} ; \quad L(t)=\frac{2}{3}(t+1)^{3 / 2}+C \\
L(0) & =0 ; \quad C=-\frac{2}{3} \\
L(t) & =\frac{2}{3}(t+1)^{3 / 2}-\frac{2}{3} ; \quad L(3)=\frac{14}{3}
\end{aligned}
$$

$$
\begin{aligned}
& 3\left\{\begin{array}{c}
2: \text { definite integral } \\
1: \text { limits } \\
1: \text { integrand } \\
1: \text { answer }
\end{array}\right. \\
& - \text { or }-
\end{aligned}
$$

## Method 2:

$3\left\{\begin{array}{l}1: \text { antiderivative with } C \\ 1: \text { solves for } C \text { using } L(0)=0 \\ 1: \text { answer }\end{array}\right.$

## 1: answer

## Method 1:

$$
\begin{aligned}
& 2\left\{\begin{array}{l}
1: 30+8 t \\
1:-\int_{0}^{t} \sqrt{x+1} d x
\end{array}\right. \\
& - \text { or - }
\end{aligned}
$$

Method 2:
$2\left\{\begin{array}{l}1: \text { antiderivative with } C \\ 1: \text { answer }\end{array}\right.$

$$
\begin{aligned}
\frac{d A}{d t} & =8-\sqrt{t+1} \\
A(t) & =8 t-\frac{2}{3}(t+1)^{3 / 2}+C \\
30 & =8(0)-\frac{2}{3}(0+1)^{3 / 2}+C ; \quad C=\frac{92}{3} \\
A(t) & =8 t-\frac{2}{3}(t+1)^{3 / 2}+\frac{92}{3}
\end{aligned}
$$

(d) $A^{\prime}(t)=8-\sqrt{t+1}=0$ when $t=63$
$A^{\prime}(t)$ is positive for $0<t<63$ and negative for $63<t<120$. Therefore there is a maximum at $t=63$.

$$
\begin{aligned}
A(t) & =30+\int_{0}^{t}(8-\sqrt{x+1}) d x \\
& =30+8 t-\int_{0}^{t} \sqrt{x+1} d x
\end{aligned}
$$

- or -

Method 2:
$3\left\{\begin{array}{c}2: \text { definite integral } \\ 1: \text { limits } \\ 1: \text { integrand } \\ 1: \text { answer }\end{array}\right.$

- or -
\{1: antiderivative with $C$
$3\left\{\begin{array}{l}1: \text { solves for } C \text { using } L(0)=0 \\ 1: \text { answer }\end{array}\right.$

Consider the curve given by $x y^{2}-x^{3} y=6$.
(a) Show that $\frac{d y}{d x}=\frac{3 x^{2} y-y^{2}}{2 x y-x^{3}}$.
(b) Find all points on the curve whose $x$-coordinate is 1 , and write an equation for the tangent line at each of these points.
(c) Find the $x$-coordinate of each point on the curve where the tangent line is vertical.
(a) $y^{2}+2 x y \frac{d y}{d x}-3 x^{2} y-x^{3} \frac{d y}{d x}=0$
$\frac{d y}{d x}\left(2 x y-x^{3}\right)=3 x^{2} y-y^{2}$
$\frac{d y}{d x}=\frac{3 x^{2} y-y^{2}}{2 x y-x^{3}}$
(b) When $x=1, y^{2}-y=6$

$$
\begin{aligned}
& y^{2}-y-6=0 \\
& (y-3)(y+2)=0 \\
& y=3, y=-2
\end{aligned}
$$

At $(1,3), \frac{d y}{d x}=\frac{9-9}{6-1}=0$
Tangent line equation is $y=3$
At $(1,-2), \frac{d y}{d x}=\frac{-6-4}{-4-1}=\frac{-10}{-5}=2$
Tangent line equation is $y+2=2(x-1)$
(c) Tangent line is vertical when $2 x y-x^{3}=0$ $x\left(2 y-x^{2}\right)=0$ gives $x=0$ or $y=\frac{1}{2} x^{2}$

There is no point on the curve with $x$-coordinate 0 .

When $y=\frac{1}{2} x^{2}, \frac{1}{4} x^{5}-\frac{1}{2} x^{5}=6$

$$
\begin{aligned}
& -\frac{1}{4} x^{5}=6 \\
& x=\sqrt[5]{-24}
\end{aligned}
$$

$2\left\{\begin{array}{l}1: \text { implicit differentiation } \\ 1: \text { verifies expression for } \frac{d y}{d x}\end{array}\right.$

$$
4 \begin{cases}1: & y^{2}-y=6 \\ 1: & \text { solves for } y \\ 2: & \text { tangent lines }\end{cases}
$$

Note: $0 / 4$ if not solving an equation of the form $y^{2}-y=k$

